ACCELERATED METHOD FOR SURFACE VIEW FACTOR EVALUATION BASED ON ERROR ESTIMATION

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ABSTRACT

An algorithm for choosing the number of quadrature nodes before calculation of a view factor is proposed. Simple criterion is introduced that allows one to estimate the error in the computed view factor. The algorithm allows one to save much computation time by always using the minimum number of nodes for each pair of surface zones and insures a desired accuracy. The algorithm is applied for model of a continuous furnace and is compared with a standard method which uses predefined number of nodes at each surface. The proposed algorithm is many times faster and also more accurate than the standard one.

Keywords: radiative heat transfer, view factors, integration

NOMENCLATURE

A – area of surface

ED – Effective distance between two quadrilaterals

F_{ij} – view (angle, configuration) factor

M – number of boundary surfaces

n – number of quadrature nodes in single integration

N – total number of quadrature nodes

$\overline{n}$ – surface normal vector

O – center of sphere

r – distance between two points

R – radius of a sphere

v – visibility of surface patches

Greek:

θ – angle between surface normal and a vector connecting two points

INTRODUCTION

The basic formula for view factor F_{12} is defined by double area integral:

$$F_{12} = \frac{1}{\pi A_1} \int_{A1} \int_{A2} \frac{\cos(\theta_1)\cos(\theta_2)}{r^2} dA_2 dA_1 , \qquad (1)$$

where A_1 and A_2 are areas of surfaces, θ_1 and θ_2 are the angles between the unit normals $\overline{n}_1$ and $\overline{n}_2$ to surface differential elements dA_1 and dA_2 and the vector, r, between those differential elements, and r is the length of that vector. This factor represents the ratio of radiant energy leaving a diffuse surface *i* that is directly incident on a diffuse surface *j* [1]. In large scale metallurgical reheating furnaces one may require evaluation of tens or even hundreds of thousands of view factors F_{ij} [2]. This then may represent a major computational effort in calculating the radiation heat transfer rates and temperatures at a large number of surface zones in a furnace. Under these circumstances accurate evaluation of a very large number (of the order of 10^4 to 10^6) view factors may become a major computational effort. The purpose of this paper is to asses *a priori* the accuracy of the view factor calculation and to develop an algorithm for choosing the integration method and number of nodes before the calculation.

CRITERIA OF ERROR ESTIMATION – EFFECTIVE DISTANCE FOR TWO POLYGONS (ED)

In this section we propose a criterion of polygon arrangement. Let us calculate a bounding sphere for each polygon. Each polygon may be inscribed into its bounding sphere, but not all polygon vertices will lie at the spherical surface. A simple algorithm for a bounding sphere calculation is available [3]. The sphere calculated is about 5% bigger than the ideal minimum-radius sphere. The algorithm is executed in two passes. The first pass finds two points that are close to maximum spaced one. This pair describes the initial guess for the sphere. The second pass compares each point to the current sphere. If the point is outside the sphere then the sphere is enlarged to include this point. Let us introduce an effective distance between two figures (polygons) as a distance between their bounding sphere centers divided by the sum of sphere radii (Fig. 1): $ED = |O_2 - O_1| / (R_1 + R_2)$.

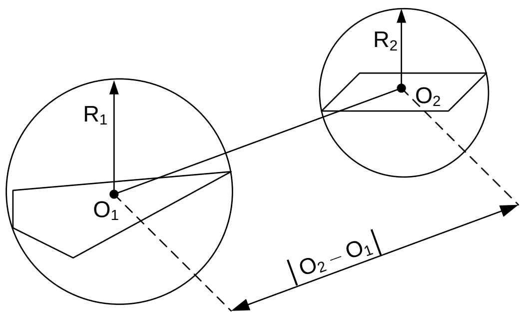

Figure 1 Geometry and nomenclature for effective distances (ED)

EXPERIMENTAL SCHEME

View factors between a number of arbitrarily located quadrilaterals were computed, and one analyzes how the accuracy of the computation depends on introduced criterion ED.

View factors between various polygons were computed using double area integration. Gaussian quadrature was used. In this section n is number of quadrature nodes used in single quadrature formula. So, the total number of nodes N is n^4 in double area integration.

Data base of quadrilaterals was generated as follows. Four basic quadrilaterals were used (Fig. 2): unit rectangle (a), a long rectangles (b), and two parallelograms with different lengths with angle 60^o (c, d). These schemes are often encountered in combustion furnace surface zone subdivision.

View factors were calculated for pairs of two quadrilaterals of the same form. The first quadrilateral was located at the plane z = 0 and was not transformed. Its area is always unit and its center is located in the origin. The second quadrilateral was generated by multiplying the size of the first quadrilateral by a «zoom factor», variation of its distance (offset) from the origin and its declination plane (Fig. 2e).

A number of zoom factors was used: 0.1, 0.5, 1.0, 2.0, 10.0. Vertical offset values are: 0.1, 0.13 … 8.7 – a geometrical progression with factor 1.3. Eighteen different values were used. The horizontal offset values are of the same progressions, but the first value is set to zero. Rotation of a polygon around the x and z axes was made. The rotation angle was varied from zero to $\pi/2$ by step of $\pi/18$. The zoom factor, the vertical and the horizontal offsets and rotation angles were varied independently. The total number of various polygon arrangements was $5x10^5$.

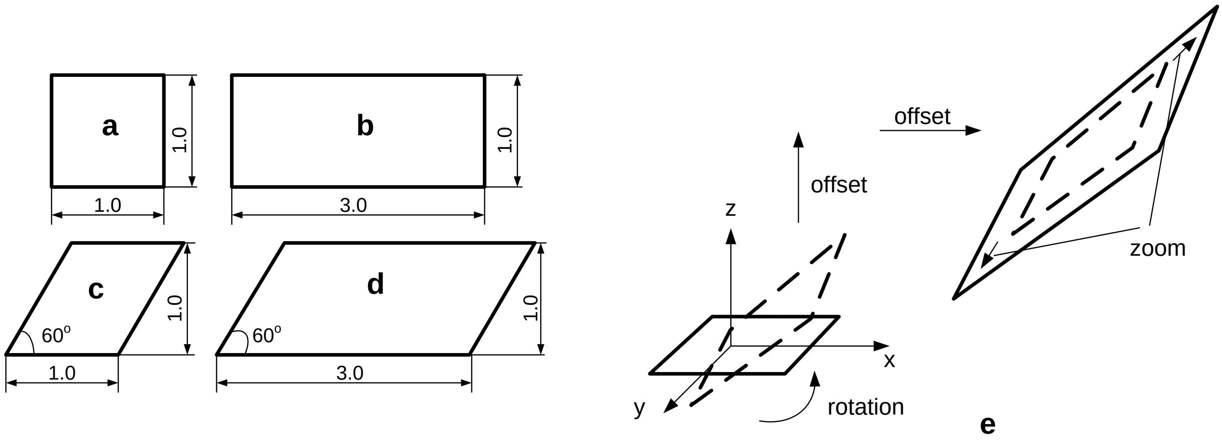

Figure 2 Quadrilaterals: a-d) basic quadrilaterals; e) transformation of second quadrilateral.

EXPERIMENTAL RESULTS

Figure 3 shows the relative error in computed view factors by double contour integration for n = 3. It can be seen that the error decreases faster as ED increases. One can find values of ED: ED(10 %), ED(5%), ED(2%) and ED(1%), such that if ED_{ij} is computed for a pair of quadrilaterals *i, j* is more than, for example, ED(5%) then the error may be estimated less than 5 % before the view factor calculation (see Fig. 3). In other words, the following estimates are carried out:

$$\begin{cases} ED_{ij} < ED(10\%) \rightarrow \Delta F_{ij} / F_{ij} < 10\% \\ ED_{ij} < ED(5\%) \rightarrow \Delta F_{ij} / F_{ij} < 5\% \\ ED_{ij} < ED(2\%) \rightarrow \Delta F_{ij} / F_{ij} < 2\% \\ ED_{ij} < ED(1\%) \rightarrow \Delta F_{ij} / F_{ij} < 1\% \end{cases}, \quad (2)$$

and so on for any desired accuracy.

Values of ED(10 %), ED(5%), ED(2%) and ED(1%) are defined for various node numbers (Table 1). Reference values of view factors were calculated by single contour integration [4] for n = 40 in double precision. All calculations were implemented in Fortran and executed on Pentium II 1.83 GHz.

Table 1 shows that the accuracy of the results strongly depends on the introduced criterion ED, and one can compute the view factors using only a few quadrature nodes. Using the estimates given in Eqs. (2) and Table 1, one can state that an algorithm of choosing the integration method and nodes number as follows:

1) Before calculation a desired accuracy *x* % for all view factors is specified. It means that a column in this table is specified.

2) For each quadrilateral its bounding sphere is calculated. View factor calculations is started next.

3) For each pair of quadrilaterals i, j criterion ED_{ij} is calculated. For each method a value n is found such that $ED(x\%, n) \le ED_{ij} \le ED(x\%, n+1)$. The left side of the inequality insures the desired accuracy. The right side of the inequality leads to the use of minimum number of quadrature nodes.

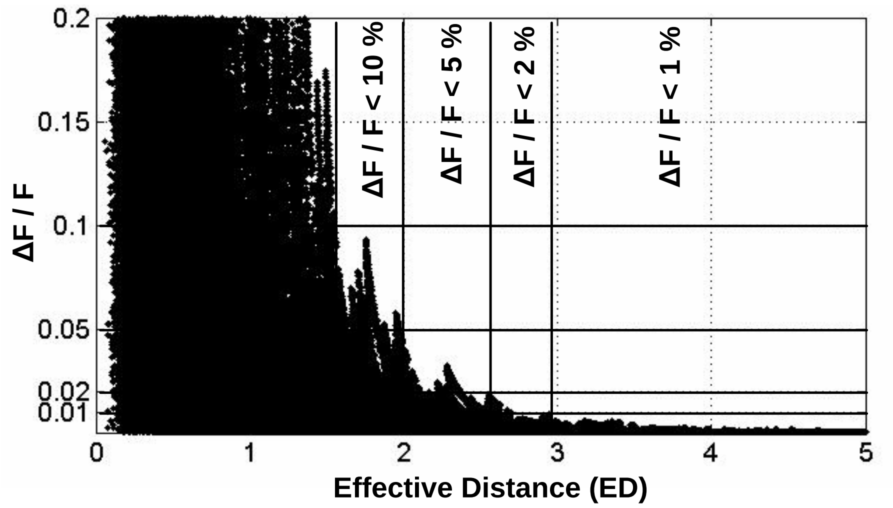

Figure 3 Dependence between relative error in computed view factors and ED. Each point represents one of $5*10^5$ view factors. Only view factors more than 10^{-4} were calculated

Table 1 Effective distances (ED) that define a relative error less than predefined value

n, number of nodes in each single integral	Nodes at each surface	ED (10 %)	ED (5%)	ED (2%)	ED (1%)
1	1^2=1	4.10	6.20	7.40	9.20
2	2^2 = 4	1.55	2.00	2.60	2.95
3	3^2 = 9	1.20	1.35	1.55	1.60
4	4^2 = 16	1.05	1.20	1.20	1.35
5	5^2 = 25	0.9	1.05	1.05	1.20
6	6^2 = 36	0.8	0.9	1.00	1.05

APPLICATION EXAMPLE

A model of continuous furnace is considered as an example of real industrial geometry with obstructions. The model is depicted in Fig. 4, and its features are the following: the number of emitting and irradiated (receiving) surfaces (walls of entire furnace and of cylindrical bars) is 3420, the number of view factors to be calculated (using the reciprocity rule) is $5.8x10^6$.

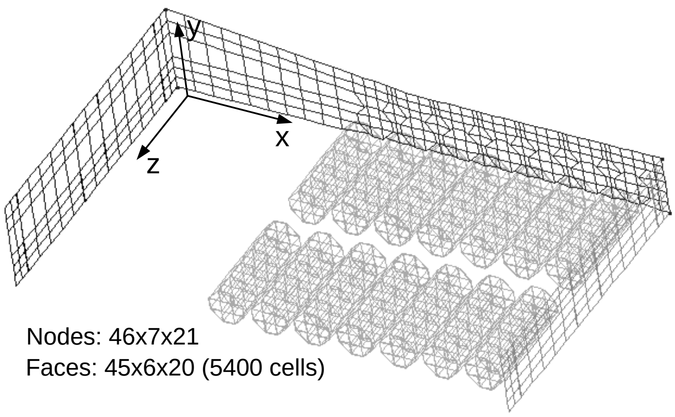

Figure 4 Model of continuous furnace in Chelyabinsk, Russia

The view factor calculation was performed using double area integration since application of this method for geometries with obstacles is straightforward. There are two schemes to take into account the obstructions: one-ray and multiple-rays scheme. In one-ray scheme [5] the view factor is multiplied by visibility term v. One ray is traced from the center of the 1-st surface towards to the center of the 2-nd surface. If the ray missed all obstructions, v = 1, otherwise v = 0. Multiple-rays scheme is more accurate, but it is also much more time-consuming. In the multiple-rays scheme [6] rays are traced between each two quadrature nodes i, j at two surfaces, and for each i, j visibility term v_{ij} is calculated. Volume-by-volume advancement algorithm was used for ray tracing [7].

RESULTS AND DISCUSSION

The calculation of view factors is not analytic, therefore the enclosing rule is not satisfied. For each row i of the MxM view factor matrix the error of enclosing can be calculated as $err_i = \left| \sum_{j=1}^{M} F_{ij} - 1 \right|$. The maximal and average errors of enclosing rule can be calculated as $err_{\max} = \max_{i=1}^{M}(err_i)$ and $err_{average} = \left(\sum_{i=1}^{M} err_i \right) \Big/ M$ respectively. Maximum and average errors of enclosing are used in the present study as cumulative values of the accuracy. These errors and time required for the view factor calculations are summarized in Tables 3 and 4. All time estimate calculations were made for a personal computer with CPU Pentium II 1.83 GHz.

Table 2 Errors and timing of calculation using a standard algorithm with predefined number of nodes at each surface

Nodes at each surface	**One-ray scheme**			**Multiple-rays scheme**		
	err_max	**err_average**	**Time, sec**	**err_max**	**err_average**	**Time, sec**
1	1.284	0.1744	48	1.284	0.1744	48
4	0.974	0.1198	53	0.960	0.1127	402
9	0.824	0.1042	67	0.800	0.0930	1929
16	0.708	0.0925	101	0.683	0.0805	6021
25	0.627	0.0835	171	0.595	0.0707	14693
36	0.561	0.0764	293	0.528	0.0631	Several hours
49	0.510	0.0706	491	0.475	0.0568	Several hours
64	0.469	0.0659	799	0.432	0.0522	Several hours
81	0.434	0.0620	1255	0.396	0.0481	Several hours
100	0.406	0.0587	1872	0.367	0.0446	Several hours

The maximum and average enclosing errors in one-ray scheme are grater respectively 5.6 % and 18 % than those in multiple-rays scheme. But time required in multiple-rays scheme is up to two orders greater. Either errors and time for calculations strongly depend on the number of nodes at each surface, but the time needed increases much more rapidly than the error decreases. For the proposed method maximum and average enclosing errors in one-ray scheme are respectively 10 %

and 21 % higher than those in multiple-rays scheme. Time required in the multiple-rays scheme is only 1.6 – 4 times greater.

Table 3 Errors and timing of calculation using proposed algorithm for choosing of nodes number

Average number of nodes at each surface	**One-ray scheme**			**Multiple-rays scheme**		
	err_max	**err_average**	**Time, sec**	**err_max**	**err_average**	**Time, sec**
1.49 (10 % acc.)	0.404	0.0585	53	0.367	0.0492	88
1.95 (5 % acc.)	0.407	0.0586	55	0.368	0.0485	132
2.33 (2 % acc.)	0.407	0.0586	57	0.368	0.0483	175
2.78 (1 % acc.)	0.406	0.0587	59	0.369	0.0479	227

It can be seen that the proposed method uses an extremely small number of nodes at each surface, but it has as good accuracy as the standard method. If one-ray scheme is used, the proposed method is 32 times faster than the standard method for identical accuracy or 2 times more accurate for the identical time required (53 sec). If multiple-rays scheme is used, the proposed method is several orders of magnitude faster than the standard method for an identical accuracy or 2.3 times more accurate for approximate identical time costs. Moreover, the proposed method combined with multiple-rays scheme is faster and more accurate than the standard method combined with the one-ray scheme.

CONCLUSIONS

The selection of many quadrature nodes for view factor calculation leads to an increase in execution time, and the minimum node selection decreases the accuracy. For each number of quadrature nodes more than 2x105 view factors were computed for various geometric arrangements. The results show that the accuracy of view factor calculation of two areas (quadrilaterals), for given number of quadrature nodes, may be estimated as function of a single parameter - effective distance ED. It allows one to select the minimum number of quadrature nodes for every pair of quadrilaterals and significantly accelerate the view factor matrix calculation with a prescribed accuracy. For two surfaces at the same ED, but at different angles (directly facing each other versus at an obtuse angle to each other), the sensitivity of the answer to the number of nodes will be quite different. Our analysis shown that 60-90 % of view factors in continuous furnace can be calculated using only 1 integration node, i.e. only 1 integrand function evaluation. This criterion is quite simple and its evaluation is as fast as evaluation of integrand function once. This is important reason for using only this single criterion instead of several criteria, because in the second case calculation of such criteria may require much more time than calculation of view factors itself.

The results presented in this paper are dependent upon the surface data base that was developed for the specific surface configuration considered. A more general distribution of surfaces, including triangular surfaces, circular surfaces, surfaces with more complex circumferences, a greater range of surface orientations, and more complex obstructed views will require the development of a new database. The validity of the pre-processing method developed here will depend upon the enlarged data base. But the surface data base is not problem-dependent and has to be generated only once.

The proposed algorithm of node number selection is applied for model of continuous furnace and compared with standard method which uses predefined number of nodes at each surface. The proposed algorithm is many times faster and also more accurate than the standard one. The proposed algorithm is universal and is also applicable for other geometries.

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REFERENCES

1. Emery, A.F., Johansson, O., Lobo, M. and Abrous, A.,1991, “A Comparative Study of Methods for Computing the Diffuse Radiation Viewfactors for Complex Structures”, J Heat Trans, Vol. 113, pp. 413-422.
2. Malikov, G., Lisienko, V., Koptelov, R., Kalugin, J. and Viskanta, R., 2010, “A numerical investigation of ray tracing method in a view factor calculation procedure for zonal radiation heat transfer in complex systems”, ASME International Mechanical Engineering Congress and Exposition, Proceedings, 9(PART A), pp. 501-505.
3. Glassner, A.S., 1990, Graphics Gems I, Academic Press, pp. 301-303.
4. Mitalas, G.P., Slephenson, D.G., 1966, “Fortran IV programs to Calculate Radiant Interchange Factors”, NRC of Canada, DBR-25, Ottawa, Canada (Report).
5. Ravishankar, M., Mazumder, S. and Sankar, M., 2010, “Application of the modified differential approximation for radiative transfer to arbitrary geometry”, Journal of quantitative spectroscopy and radiative transfer, 111(14), pp. 2052-2069.
6. Walton, G.N., 2002, “Calculation of obstructed view factors by adaptive integration, National Institute of Standards and Technology”, NISTIR 6925 (Report).
7. Mazumder, S., 2008, “Methods to accelerate ray tracing in the Monte Carlo method for surface-to-surface radiation transport”, J. Heat Trans, 128(9), pp. 945-952.

CAPTION LIST

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