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Calculation of Exchange Areas in Models of High-Temperature Radiant Systems

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Abstract—A method based on the discrete ~~ordinates~~ model is proposed for the determination of direct exchange areas in zonal calculations of radiant heat transfer. This algorithm proves faster than numerical integration in assessing the matrix of direct exchange areas corresponding to the interaction of surface and internal (volume) zones. As a test case, the precision and speed of the proposed method are compared with those of numerical integration in simulating the heat transfer within the IFRF experimental furnace.

Keywords: radiant heat transfer, zonal method, direct exchange areas, DEA matrix, discrete ordinates model
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The simulation of radiant heat transfer in furnaces for the heat treatment of metal is important in their design and operation. Several approaches have been developed for this purpose. Commercial software for the calculation not only of radiant heat transfer but also of hydrodynamics, combustion, and other physical processes generally employ finite-element methods—for example, methods based on finite volumes. In those methods, all types of energy transfer within the system are calculated by means of a relatively fine grid, with rigorous boundary conditions. Another approach is to divide all the surfaces in the system and its working volume into some number of relatively large zones, with the determination of the matrix $\overline{ss}$ of direct exchange areas (in the case of nonreflecting zones) or general exchange areas (in the case of reflecting zones). The elements $\overline{s_i s_j}$ of this matrix permit relatively simple calculation of the heat flux exchanged by zones s_i and s_j. In the present work, we consider direct exchange areas (DEA) [1].

EXISTING METHODS OF DETERMINING THE DEA MATRIX

In practical problems, the main method of determining the elements of the DEA matrix is double numerical integration with respect to the area or volume [1]. Obviously, in this approach, a considerable volume of calculations is required on account of the large number of surface and internal (volume) zones in a complex geometric configuration. In addition, the path of each beam in the system must be traced until it intersects with the closest opaque surface. That also takes considerable time.

The Monte Carlo method has also been used to calculate the DEA matrix [2]. However, numerous statistical tests are required to ensure sufficient precision of the results. That again increases the computational burden.

CALCULATION OF THE DEA MATRIX BY THE DISCRETE ~~ORDINATES~~ MODEL

The proposed method of calculating the DEA matrix reduces the number of beams to be traced because information gathered in calculating some elements of the DEA matrix (of surface–surface type) is used in determining other matrix elements (of surface–volume and volume–volume type). This approach was first developed for the finite-element method by Lockwood [3]; earlier, it was developed by 1
Lisienko for the Monte Carlo method [2, 4].

In the zonal method, the direction of the radiation is selected on the basis of the configuration of the zones in the system. To improve the computational precision, it is expedient to divide all the surface zones in the system into relatively small sections (subzones) and to select the line connecting the geometric centers of those subzones as the direction of radiant propagation.

In Fig. 1, we show an example of such division for the two-dimensional case.

The surface zones $s_1, s_2, s_3, \ldots, s_i$ and volume zones $g_1, g_2, g_3, \ldots, g_k$ are identified in the system. Suppose that surface zone s_i is divided into M_i subzones, denoted by $s_i^1, s_i^2, \ldots, s_i^m, \ldots, s_i^{M_i}$. Analogously, suppose that zone s_j is divided into N_j subzones s_j^n, $n =$

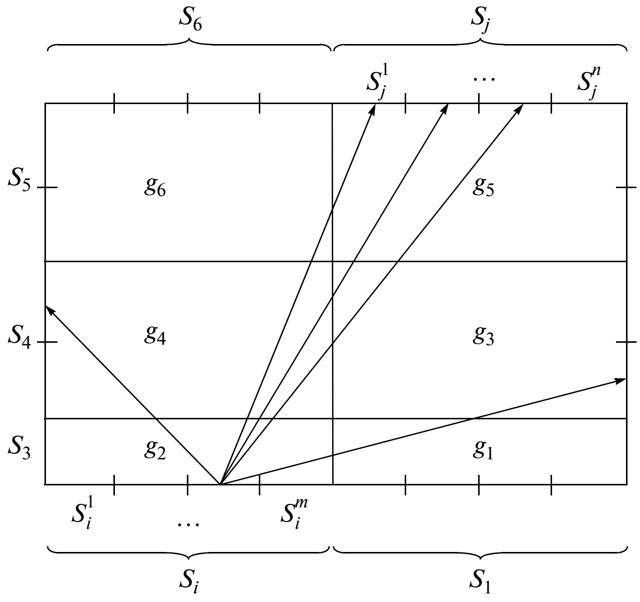

Fig. 1. Example of the creation of surface and volume zones in a system.

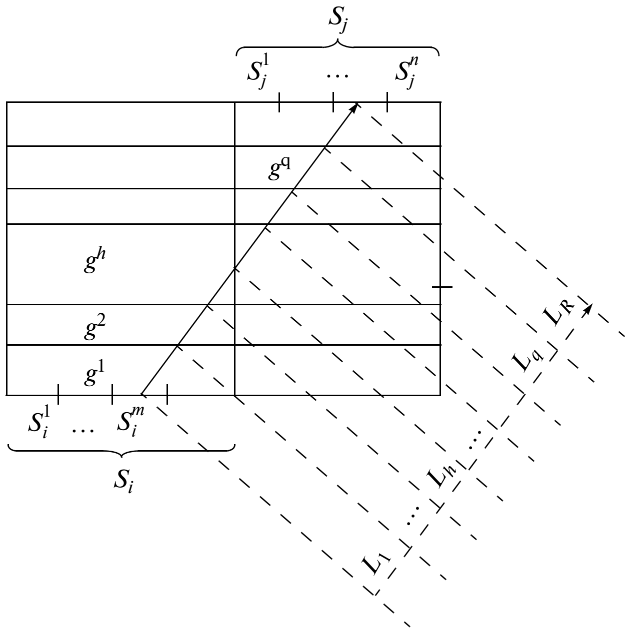

Fig. 2. Finding the elements of the DEA matrix by the discrete ~~ordinates~~ model.

1, … N_j. We assume that the whole radiant flux from subzone s_i^m to subzone s_j^n is emitted along the direction connecting their geometric centers.

To obtain the exchange surface of two surface zones s_i and s_j (Fig. 1), we need to calculate the sum of terms of the form $\overline{s_i^m s_j^n}$ ($m = 1, \ldots, M_i$; $n = 1, \ldots, N_j$)

$$\overline{s_i s_j} = \sum_{m_i=1}^{M_i} \sum_{n_j=1}^{N_j} \overline{s_i^m s_j^n}.$$

For each pair of subzones s_i^m and s_j^n, we calculate the values of k^* and L^* that best satisfy the relation

$$\overline{s_i^m s_j^n} = e^{-k^* L^*} \int\limits_{A_{m_i}} \int\limits_{A_{nj}} \frac{\cos\theta_i \cos\theta_j}{\pi L^2} dA_{n_j} dA_{m_i} = e^{-k^* L^*} A_{m_i} \psi_{m_i n_j},$$

where $\psi_{m_i n_j}$ is the inclination of subzone s_i^m from subzone s_j^n when there is a transparent medium between them.

We calculate $k^* L^*$ from the formula

$$k^* L^* = k_1 L_1 + k_2 L_2 + \ldots + k_r L_r + \ldots + k_R L_R = \sum_{r=1}^{R} k_r L_r.$$

Here k_r is the absorption coefficient of the radiation in the volume zone on the beam path from s_i^m to s_j^n; L_r is the path length of the beam in the volume zone; R is the number of volume zones that the beam intersects on its path.

Thus, the direct exchange area of two surface zones may be found from the formula

$$\overline{s_i s_j} = \sum_{m_i=1}^{M_i} \sum_{n_j=1}^{N_j} e^{-\sum_{r=1}^{R} k_r L_r} A_{m_i} \psi_{m_i n_j}.$$

The elements of the surface–volume DEA matrix may expediently be found in parallel with tracing the beams connecting surface zones.

We denote by superscript $r = 1, \ldots, R$ the volume zones intersected by the beam on its path from subzone s_i^m to subzone s_j^n. In other words, if the beam intersects zones g_2, g_3, g_4, and g_5 on its path (as in Fig. 1), these zones will be assigned the new notation g_1, g_2, g_3, and g_4 for the subsequent discussion (Fig. 2).

Suppose that the beam intersects zone g^h ($1 \le h \le R$) in its path and remains in this zone over a distance L_h. In that case, the flux absorbed by zone g^h is determined as the part of the initial flux from subzone s_i^m to subzone s_j^n corresponding to the formula

$$Q_m = E_b A_{m_i} \psi_{m_i n_j},$$

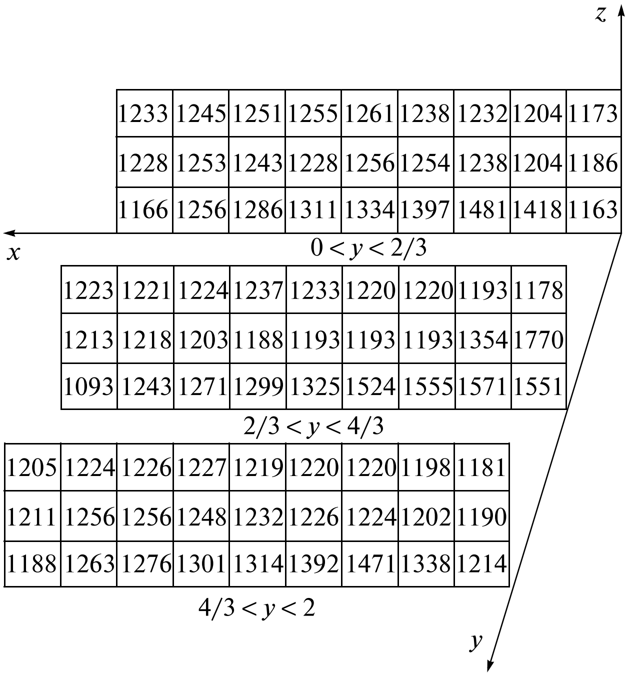

Fig. 3. Measured temperatures (K) of gas zones within the IFRF experimental furnace.

where A_{m_i} is the area of cell (subzone) s_i^m; E_b is the radiant intensity of an absolutely black body.

According to the Bouguer law, the proportion Q_m of the radiation absorbed by zone g^h is

$$g_{\text{abs}} = \frac{Q_m e^{-\sum_{r=1}^{h-1} k_r L_r} (1 - e^{-k_h L_h})}{Q_m}.$$

Hence, the contribution that the radiation from subzone s_i^m to subzone s_j^n makes to the direct exchange area of zones s_i and g^h will be

$$\overline{s_i g^h_{s_i^m \to s_j^n}} = \frac{Q_m \gamma_{\text{abs}}}{E_b} = A_{m_i} \Psi_{m_i n_j} e^{-\sum_{r=1}^{h-1} k_r L_r} (1 - e^{-k_h L_h}).$$

If we add all the contributions with respect to n, m, and j, we obtain the total direct exchange area of zones s_i and g^h

$$\overline{s_i g^h} = \sum_{j=1}^{J} \sum_{m_i=1}^{M_i} \sum_{n_j=1}^{N_j} \overline{s_i g^h_{s_i^m \to s_j^n}}$$

$$= \sum_{j=1}^{J} \sum_{m_i=1}^{M_i} \sum_{n_j=1}^{N_j} A_{m_i} \Psi_{m_i n_j} e^{-\sum_{r=1}^{h-1} k_r L_r} (1 - e^{-k_h L_h}).$$

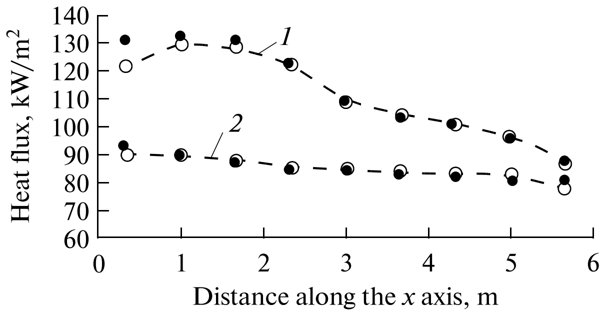

Fig. 4. Results of simulating the radiant heat transfer in the IFRF experimental furnace by numerical integration (●) and by the discrete ordinates model (○): (*1*) heat flux at the furnace floor; (*2*) heat flux at the furnace roof.

Thus, the direct exchange area of surface zone s_i and volume zone g^h is calculated in parallel with tracing the beams that connect surface subzones to other surface subzones and does not require numerical integration over the volume and surfaces of zones.

By analogy with the foregoing, we may obtain formulas for the exchange areas between two volumes.

As a test calculation, we consider a system used by several researchers [5, 6]: a rectangular region simulating the experimental furnace designed by the International Flame Research Foundation (IFRF). The furnace dimensions are 6 × 2 × 2 m. The absorption coefficient within volume zones is assumed to be 0.2 m^{-1}. The temperatures of the hearth, roof, and walls, are 320, 1090, and 1090 K, respectively; the emissivity is 0.86, 0.70, and 0.70, respectively.

We consider three zones along the short sides of the furnace, and ten zones along the long sides. The total number of surface zones is 126; the total number of volume zones is 81. The measured temperatures in the volume zones are presented in Fig. 3.

The results obtained by different modeling methods for the incident flux on the furnace roof and floor are compared in Fig. 4.

The mean discrepancy between the results obtained by the two methods is 1.42% for the hearth and 1.12% for the roof. The mean relative error in determining the exchange areas is 2.9% for numerical integration and 0.07% for the discrete ordinates model. Comparative speed data are as follows (numerical integration/discrete ordinates model):

	Number of surface zones	
56	126	224
	Calculation time. s	
2.2/0.3	58.6/2.6	624.9/14.4
	Number of beams (thousands)	
87.0/3.1	666.0/16.0	2963.0/50.0

CONCLUSIONS

We have proposed a new method for determining the direct exchange areas in zonal calculations of radiant heat transfer. The new method is based on the discrete ordinates model. In this approach, all the elements of the DEA matrix may be determined by tracing five beams between surface zones. This method considerably reduces the calculation time. The precision of the results in the discrete ordinates model is comparable to that in numerical integration, but the calculation time is reduced by more than an order of magnitude.

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