LOWER-MODULAR ELEMENTS OF LATTICE OF EPIGROUP VARIETIES

A semigroup S is called an *epigroup* if for any element x of S there is a natural n such that x^n is a *group element* (this means that x^n lies in some subgroup of S). An extensive information about epigroups may be found in [4]. It is natural to consider epigroups as unary semigroups, that is semigroups with an additional unary operation. In particular, this allows to consider varieties of epigroups as algebras with the two mentioned operations.

Taking into consideration the study of [2] we are covering the subject of special elements in the lattice of varieties epigroups. Neutral, modular and upper-modular elements this lattice (the definitions all types of special elements may be founded in [1]) are considered in [2]. In this text we consider lower-modular elements the lattice of varieties epigroups.

An element x of a lattice $\langle L; \vee, \wedge\rangle$ is called *lower-modular* if $\forall y, z \in L \quad x \leq y \longrightarrow x \vee (y \wedge z) = y \wedge (x \vee z)$.

Lower-modular elements the lattice of varieties semigroups are considered in [3]. In this text there is epigroup analogs obtained results.

We denote by $\mathcal{T}$ and $\mathcal{SL}$ the trivial variety and the variety of all semilattices. Recall that a variety is called 0-*reduced* if it is defined by 0-reduced identities only.

Theorem 1. *A epigroup variety $\mathcal{V}$ is lower-modular if and only if $\mathcal{V} = \mathcal{M} \vee \mathcal{N}$ where $\mathcal{M}$ is one of the varieties $\mathcal{T}$ or $\mathcal{SL}$ and $\mathcal{N}$ is a 0-reduced variety.*

References

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