Many-Body Renormalization of the Minimal Conductivity in Graphene

F. Guinea[1] and M. I. Katsnelson[2,3]
[1]Instituto de Ciencia de Materiales de Madrid, CSIC, Sor Juana Ines de la Cruz 3, 28049 Madrid, Spain
[2]Radboud University Nijmegen, Institute for Molecules and Materials, Heyndaalseweg 135, NL-6525AJ Nijmegen, The Netherlands
[3]Department of Theoretical Physics and Applied Mathematics, Ural Federal University, Mira Street 19, 620002 Ekaterinburg, Russia
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The conductance of ballistic graphene at the neutrality point is due to coherent electron tunneling between the leads, the so called pseudodiffusive regime. The conductance scales as a function of the sample dimensions in the same way as in a diffusive metal, despite the difference in the physical mechanisms involved. The electron-electron interaction modifies this regime, and plays a role similar to that of the environment in macroscopic quantum phenomena. We show that interactions can change substantially the transport properties. In the presence of nearby metallic layers, the conductance near the neutrality point can decrease with decreasing temperature, and reach values well below the quantum unit of conductance, as in an insulator.

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The electrical conductivity of solids is determined by electron or hole excitations at the Fermi level. One of the most striking features of graphene is its finite metallic conductivity when the Fermi surface shrinks to a point, and the density of charge carriers vanishes [1,2]. The origin of this minimal conductivity is a problem of fundamental relevance. Early experiments suggested that the conductivity at the neutrality point was of the order of a conductance quantum, while recent measurements in high mobility samples give a much lower value [3,4]. Carriers become localized when the conductivity drops below the quantum unit, but in graphene localization is suppressed by "Klein" tunneling [5]. Calculations show that graphene remains metallic at the neutrality point. The same conclusion can be reached assuming that graphene is defect free and ballistic at the neutrality point, due to an essentially quantum phenomenon, transmission via evanescent waves [6–8]. We analyze here the effect of the electron-electron interaction in this regime, and, thus, on the minimal conductivity of graphene. The experimental setups envisaged in this work are sketched in Fig. 1.

Experiments show that the Coulomb interaction between electrons changes substantially the electronic properties near the Dirac point in high mobility suspended systems [10]. The effect of interactions on the conductivity of graphene at the Dirac point has been addressed theoretically, using diagrammatic methods and starting from the Kubo expression for the conductivity [11–14]. The conclusion of these works is that the metallic nature of graphene near the Dirac point is not changed by interactions. We consider here the alternative description where the conductance of a ballistic graphene sample is studied using Landauer's formalism, adding later the electron-electron interaction, and come to essentially different conclusions. It turns out that the interaction effects suppress essentially the transport via evanescent waves leading to temperature (or sample-size) dependent minimal conductivity, in agreement with recent experimental observations [4].

Conduction in a perfect ballistic graphene sample at the Dirac point is due to tunneling of electrons with well defined momentum parallel to the direction of current [6,7]. The summation of the transmission coefficients of all these parallel momentum channels gives rise to a conductance inversely proportional to the system length, defined as the transport direction, and inversely proportional to the width of the ribbon. This scaling with the sample dimensions is the same as in a diffusive metal, leading to the term "pseudodiffusive regime" [7]. This approach can be generalized to graphene bilayers [15–17], and to samples of arbitrary shapes [18,19]. We assume that the tunneling electrons can excite electron-hole pairs and other electronic excitations of the system, which are considered to be independent degrees of freedom. This approach can be justified by replacing the excitations of the electronic system by bosons, each of which is weakly coupled to the tunneling electron [20–22]. This analysis is closely related to other approaches to the study of transport properties in the presence of an environment [23,24]. In general, transport through an interacting system can be formulated in terms of Green's functions [25]. The scheme used here gives an exponential renormalization to the noninteracting Green's function, which is equivalent to the resummation of diagrams performed within the RPA (see the Supplemental Material [26]).

We analyze transport through a rectangular graphene sample of dimensions L_x, L_y, where the x axis is the current direction, and $L_x, L_y \gg a$, where a is the lattice spacing, see Fig. 2. For simplicity we use periodic boundary conditions along the y direction [6]. We assume that the

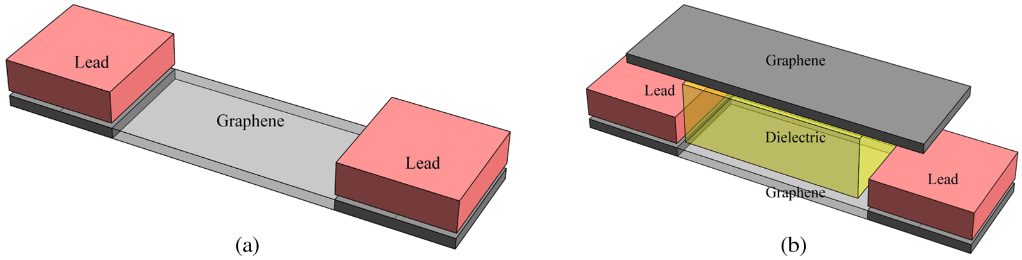

FIG. 1 (color online). Sketch of the devices studied in the text (see also Ref. [9]). (a) Transport in ballistic neutral graphene. Carriers are injected from heavily doped regions below the contacts. (b) As in (a) with an additional metallic layer (graphene with a finite carrier concentration) separated by a dielectric (BN).

wave function is coherent along the y direction and the transverse momentum is quantized, $k_y = 2\pi n/L_y$, where n is an integer. Thus, the problem is reduced to a set of one-dimensional (1D) problems, in a similar way to the noninteracting case. This approach neglects scattering between different channels mediated by the interactions. As discussed below, we obtain an exponential renormalization of the transport amplitudes. Interchannel scattering will lead to corrections that, within perturbation theory, will be smaller. The same arguments can be used to neglect the logarithmic renormalization of the Fermi velocity near the neutrality point. Note also that this effect is further suppressed by the presence of a dielectric, see Fig. 1.

The tunneling along the x direction is studied by estimating the optimal path in imaginary time, and adding to the action along that path the corrections due to the interactions with the environment. The barrier through which tunneling takes place is $V(x) = \hbar \mathrm{v}_F k_y$ for $0 \leq x \leq L_x$, and v_F is the Fermi velocity. The path under the barrier is simply $x(\tau) = \mathrm{v}_F \tau$, with $0 \leq \tau \leq L_x/\mathrm{v}_F$.

The tunneling amplitude, in the absence of interaction effects, is $T_0(k_y) \cong e^{-k_y L_x}$. The correction to the action due to the interactions with the environment can be written as (see the Supplemental Material [26])

$$\delta S = \frac{1}{2}\int_{-\infty}^{+\infty} d\tau \int_0^{\beta} d\tau \times \int_{-\infty}^{+\infty} \frac{dq}{2\pi} e^{iq[x(\tau)-x(\tau')]} v_q^2 \langle \hat{T}[\rho_q(\tau)\rho_{-q}(\tau)]\rangle \quad (1)$$

where $\beta = 1/T$ is the inverse temperature, v_q is the Fourier component of the Coulomb interaction, ρ_q is the electron density operator, and $\hat{T}$ is the time-ordering operator. Using the fluctuation-dissipation theorem and proceeding further as in Ref. [22], we come to the expression

$$\delta S = \frac{1}{2}\int_{-\infty}^{+\infty} d\tau \int_0^{\beta} d\tau \int_{-\infty}^{+\infty} \frac{dq}{2\pi} \times \int_{-\infty}^{+\infty} d\omega\, e^{iq[x(\tau)-x(\tau')-\omega|\tau-\tau'|]} W(q,\omega), \quad (2)$$

where $W(q,\omega)$ is a density of states that includes the density of states of modes in the environment, and their coupling to the tunneling electron

$$W(q,\omega) = \frac{v_q}{\epsilon(q,\omega)}, \quad (3)$$

and $\epsilon(q,\omega)$ is the dielectric function. The tunneling amplitude is finally $T(k_y) \cong T_0(k_y) e^{-\delta S}$.

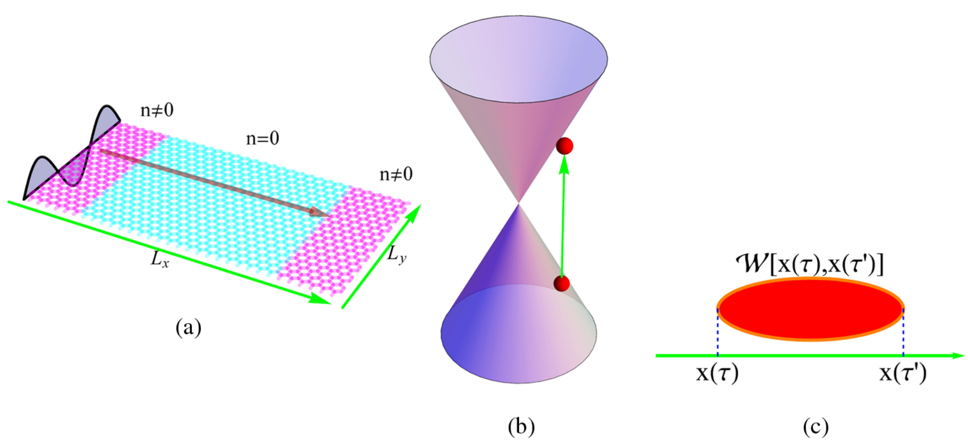

FIG. 2 (color online). (a) Sketch of the processes considered in the text. An electron wave packet, coherent in the direction normal to the direction of the current, is transferred between two electrodes. (b) The tunneling process is accompanied by the emission of electron-hole pairs. (c) Lowest order diagram that describes the process.

For the effective 1D problem defined here, we have (see the Supplemental Material [26])

$$v_q \simeq \begin{cases} -\frac{2e^2}{\epsilon_0}\log(qL_y), & qL_y \ll 1 \\ \frac{2\pi e^2}{\epsilon_0 q L_y}, & 1 \ll qL_y, \end{cases} \tag{4}$$

where ϵ_0 is the dielectric constant of the environment. This expression interpolates between the expected 1D behavior for $qL_y \ll 1$, and the 2D Coulomb interaction, normalized to the width of the sample for $qL_y \gg 1$.

We consider first an environment made up of the electron-hole excitations of graphene at the neutrality point. The dielectric function can be written as $\epsilon(q,\omega) = 1 + v_q\chi_{1D}(q,\omega)$, where $\chi_{1D}(q,\omega)$ is the polarization function of our 1D problem. The dielectric function of a graphene ribbon was calculated in Ref. [24]. In wide ribbons, $L_y \gg a$, the leading contribution to the transmission comes from evanescent states with decay length $\lambda \approx L_y$. The relevant momenta in the screened interaction, Eq. (3), are $q \approx L_y^{-1}$. Then, the Coulomb potential does not mix subbands [24], and we can approximate [27]

$$\chi_{1D}(q,\omega) \approx L_y\chi_{2D}(q,\omega) \approx L_y \frac{q^2}{\sqrt[4]{\mathrm{v}_F^2 q^2 - \omega^2}}. \tag{5}$$

A more detailed derivation of this equation can be found in Eq. (S3) in the Supplemental Material [26].

In the ballistic regime, where $x(\tau) = \mathrm{v}_F\tau$, the time integrals in Eq. (2) can be reduced to

$$\begin{aligned} \delta S_G &\approx \frac{L_x}{(2\pi)^2 L_y \mathrm{v}_F}\int_{L_x^{-1}}^{a^{-1}} dq \int_{\mathrm{v}_F q}^{\mathrm{v}_F a^{-1}} d\omega \int_0^{L_x/\mathrm{v}_F} d\tau \frac{4(2\pi e^2/\epsilon_0)^2}{16(\omega^2 - \mathrm{v}_F^2 q^2)^2 + (2\pi e^2/\epsilon_0)^2 q^2} e^{iq\mathrm{v}_F\tau} e^{-\omega\tau} \\ &\approx \frac{L_x}{(2\pi)^2 L_y \mathrm{v}_F}\int_{L_x^{-1}}^{a^{-1}} dq \int_{\mathrm{v}_F q}^{\mathrm{v}_F a^{-1}} d\omega \frac{4(2\pi e^2/\epsilon_0)^2}{16(\omega^2 - \mathrm{v}_F^2 q^2)^2 + (2\pi e^2/\epsilon_0)^2 q^2} \frac{\omega}{\omega^2 + \mathrm{v}_F^2 q^2} \\ &\approx \frac{L_x}{8\pi L_y} \frac{\alpha^2}{4\sqrt{2} + \alpha} \log\left(\frac{L_x}{a}\right), \end{aligned} \tag{6}$$

where $\alpha = (2\pi e^2)/(\epsilon_0 \mathrm{v}_F)$, and we have kept only the leading term in L_x/a. The subindex G stands for the graphene contribution to the effective action.

We now consider the changes induced in the environment by the presence of a metallic layer. We describe the metal in terms of its density of states $v_{1D} \approx L_y v_{2D}$, Fermi velocity v_F^M, Fermi energy ϵ_F, Fermi momentum k_F, mean free path ℓ, and diffusion coefficient $D = (\mathrm{v}_F^M \ell)/2$. The polarizability of the metal, for $\omega \le \epsilon_F$ and $q \le k_F$, can be approximated by

$$\chi_{1D}^M(q,\omega) \approx \begin{cases} \frac{v_{1D} D q^2}{i\omega + Dq^2}, & q \le \ell^{-1} \\ \frac{v_{1D}\mathrm{v}_F^M q}{i\omega + \mathrm{v}_F^M q}, & \ell^{-1} \le q \le k_F. \end{cases} \tag{7}$$

In the RPA approximation, the retarded interaction is given, approximately, by $W(q,\omega) \approx [\chi_{1D}^M(q,\omega)]^{-1}$. Then, the value of δS_M can be divided into a diffusive and a ballistic contribution

$$\delta S_M = \delta S_d + \delta S_b \approx \frac{L_x^2}{4\pi g \ell L_y} + \frac{L_x}{8\pi L_y}\log(g), \tag{8}$$

where $g = k_F \ell$ is the conductivity of the metallic layer.

At finite temperatures, $T \neq 0$, low energy modes of the environment are highly excited, so that their coupling to the tunneling particle can be neglected. The absence of low energy transitions mediated by the environment can be rewritten as the existence of a new lower cutoff in the momentum transfer between the particle and the environment, $q_c \approx \max(L_x^{-1}, T/\mathrm{v}_F)$, which replaces L_x^{-1} in Eq. (6). Figure 3 shows the temperature dependence of the resistivity taking into account a neutral graphene environment δS_G in Eq. (6), and a metallic environment δS_M in Eq. (8). The parameters used for the metallic layer are appropriate for graphene away from the neutrality point. For this choice of parameters, the final conductivity is mostly determined by the contribution from the diffusive modes of the metal.

The pseudodiffusive regime can be generalized to situations with external magnetic fields [28]. The presence of a magnetic field changes the conductivity in the metal, due to the suppression of coherence effects. In addition, the classical trajectories in the neutral ballistic graphene layer are modified on scales comparable to the magnetic length ℓ_B. A simple perturbative estimate of the self-energy in the presence of a magnetic field shows that the effective interaction is modified (see the Supplemental Material [26]):

$$W(q,\omega) \approx \int dq e^{-(q-q')^2/\ell_B^2} W(q,\omega).$$

This B dependent broadening suggests the use of the lower cutoff $q_c \approx \max(L_x^{-1}, T/\mathrm{v}_F, \ell_B^{-1})$. The magnetic field dependence of the inverse conductance using this approximation is also shown in Fig. 3. Note that a numerical constant c in the definition of q_c will change the temperature and magnetic field scales, although not the qualitative trends.

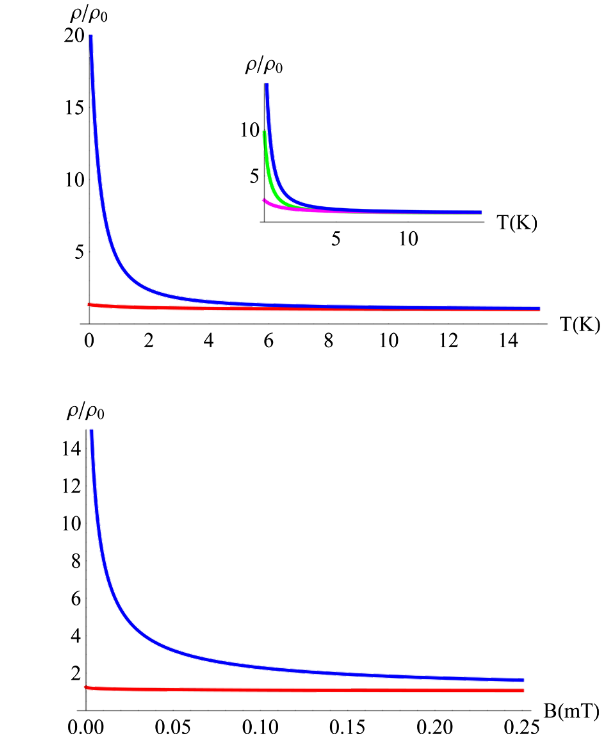

FIG. 3 (color online). Top: temperature dependence of the inverse conductance, normalized to the noninteraction value $\sigma_0 = e^2/(\pi\hbar)$ for $L_x = 4\mu$, $L_y = 1\mu$. Red (upper line): contribution from the graphene excitations δS_Gg Eq. (6). Blue (lower line): contribution from a metallic layer δS_M, Eq. (8). The two terms that describe the contribution from the metal, δS_d and δS_b, are shown in the inset. Green (middle line): diffusive part δS_d in Eq. (8). Magenta (lowest line): ballistic part δS_b in Eq. (8). The carrier density in the metal is $n = 10^{11}$ cm^{-1}, and the elastic mean free path is $\ell = 100$ nm. Bottom: magnetic field dependence of the inverse conductance for $T = 1$ K. The remaining parameters are the same as in the top figure.

Another situation where electron tunneling is relevant is ballistic transport through a p-n junction [29,30]. The properties of a planar p-n junction are determined by the electric field ε when the potential lies close to the Dirac energy, $V(x) \approx e\varepsilon x$. Electrons with a well defined parallel momentum k_y and a dispersion $\varepsilon_k = \mathrm{v}_F\sqrt{k_x^2 + k_y^2}$ have a gap of forbidden energies $\Delta_{k_y} = \mathrm{v}_F k_y$. Hence, an electron with momentum k_y must tunnel through a barrier through the region $-(\mathrm{v}_F k_y)/(2\varepsilon) \leq x \leq (\mathrm{v}_F k_y)/(2\varepsilon)$. The probability of tunneling is [8,29] $T_0(k_y) \approx e^{-(\mathrm{v}_F k_y^2)/\varepsilon}$. Interactions suppress tunneling through p-n junctions in the manner discussed above, with the replacement $L_x \leftrightarrow (\mathrm{v}_F k_y)/\varepsilon$ in Eqs. (6) and (8). For example, instead of Eq. (6) we have

$$T(k_y) \approx e^{-\frac{\mathrm{v}_F k_y^2}{\varepsilon}} \left(\frac{\mathrm{v}_F|k_y|}{a\varepsilon}\right)^{(\mathrm{v}_F|k_y|\alpha^2/8\pi L_y \varepsilon 4\sqrt{2}+\alpha)}. \quad (9)$$

This renormalization changes essentially the angular dependence of the tunneling probability for very small

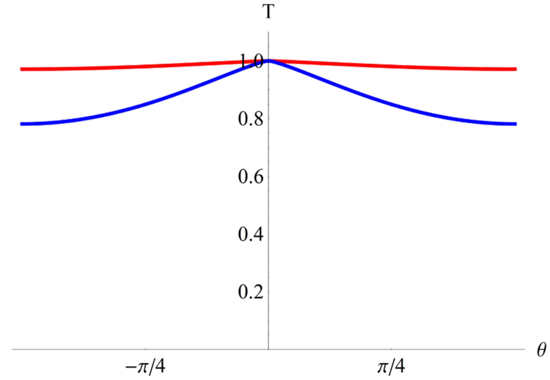

FIG. 4 (color online). Angular dependence of the correction due to interactions in a p-n junction. The p-n junction has a length of 1 nm and separates two regions of densities $n = \pm 10^{12}$ cm^{-2}. The value of α is 1.1. Red (upper line): intrinsic effect in graphene at the neutrality point δS_G, Eq. (6). Blue (lower line): effect of a metallic layer, δS_M, Eq. (8). The charge density in the metal is $n = 10^{13}$ cm^{-2}.

angles, $|k_y| \ll \alpha^2/(8\pi L_y)$. At the same time, $T = 1$, an exact property for normal incidence [8], remains unchanged when the electron-electron interactions are taken into account. The changes induced in the angular dependence of the transmission are shown in Fig. 4.

The dependence of the total conductance, $\propto L_y/(2\pi)\times \int dk_y T(k_y)$, on the electric field is changed, due to the renormalization in Eq. (9), from $\sqrt{\varepsilon}$ (which corresponds to the Schwinger effect, with the pair intensity production $P \propto \varepsilon G \propto \varepsilon^{3/2}$, see Refs. [31,32] and references therein) to $P \propto \varepsilon^2$. The crossover takes place at the electric field $\varepsilon \approx (\alpha^2 \mathrm{v}_F)/(8\pi L_y^2)$.

Tunneling between localized states is another mechanism that gives rise to a finite conductivity of graphene at the neutrality point [33]. The interaction effects discussed here will also influence this mechanism [22].

The lack of an intrinsic limit to the conductivity of ballistic graphene at the neutrality point suggests new ways to manipulate its value. The combination of quantum tunneling and interactions implies that ballistic graphene at the neutrality point can be used to study dephasing processes under a variety of external probes.

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